

## PARAMETRIC EQUATIONS

If two related variables depend on a third variable,  $t$ , then

$$x = f(t) \text{ and } y = g(t)$$

are **Parametric Equations**.

$t$  is the independent variable and is called the **parameter**.

Parametric equations can be used to separate horizontal ( $x$ ) and vertical ( $y$ ) motion along a curve. The third variable,  $t$ , is often time as in the following example, but can also be an angle.



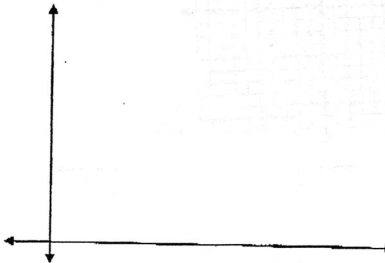
1. A golfer hits a golf ball with an initial velocity of 130 ft/sec at a  $45^\circ$  angle of elevation. The following parametric equations model the path of the ball. (We will learn to derive these later.)

$$x = 92t$$

$$y = -16t^2 + 92t$$

Make a table of values to represent the path of the curve for  $t$  in  $[0, 5]$ . Then plot the points from the table onto the graph.

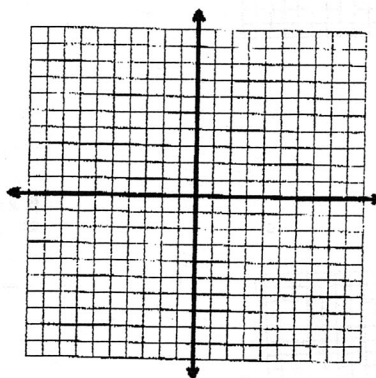
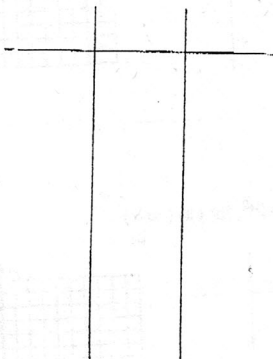
$t$	$x$	$y$



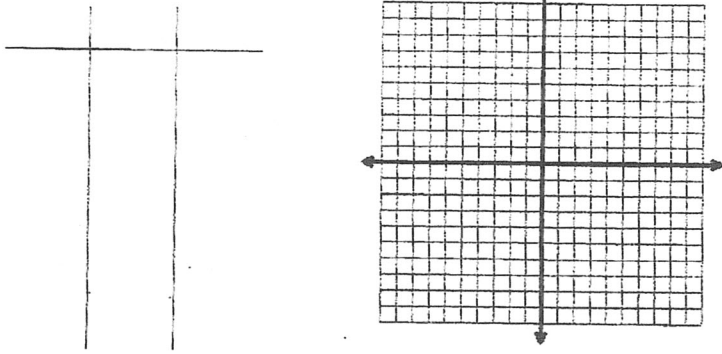
For each pair of parametric equations:

- Make a table of values
- Draw the graph and show the orientation.
- Write the rectangular equation.

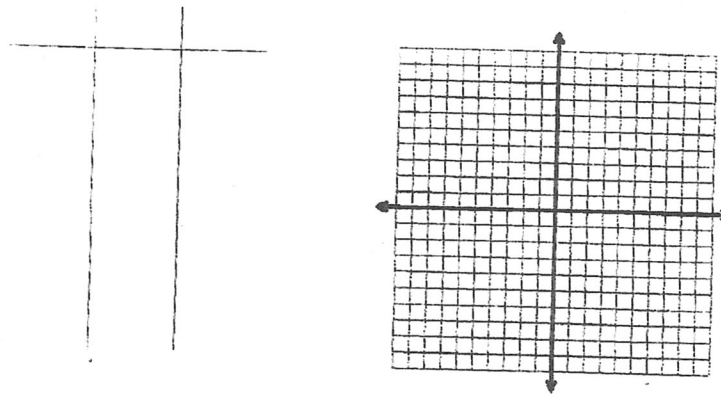
2.  $x = 1 - 2t$ ,  $y = 2 + 2t$  for  $t$  in  $[0, 4]$



3.  $x = t - 2, y = t^2$  for  $t$  in  $[-1, \infty)$



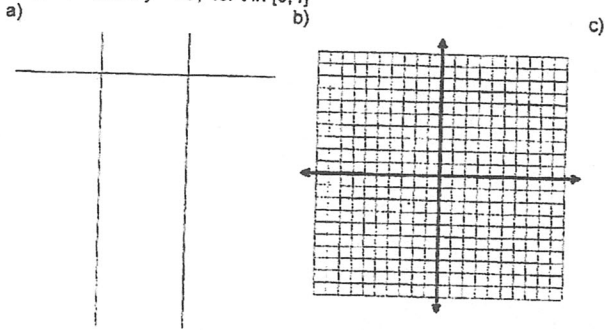
4.  $x = \sqrt{t} + 4, y = \sqrt{t} - 4$  for  $t$  in  $[0, \infty)$



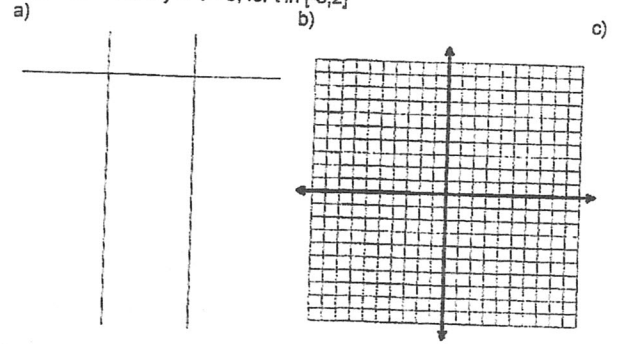
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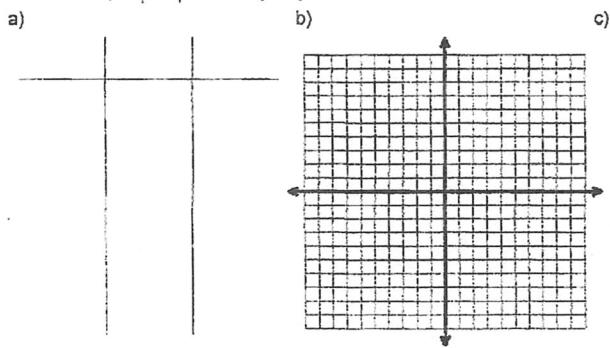
1.  $x = 1 + t$  and  $y = 2t$ , for  $t$  in  $[0, 4]$



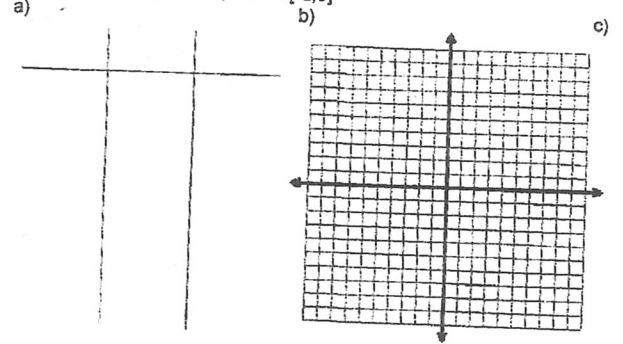
2.  $x = -2t + 1$  and  $y = t - 5$ , for  $t$  in  $[-3, 2]$



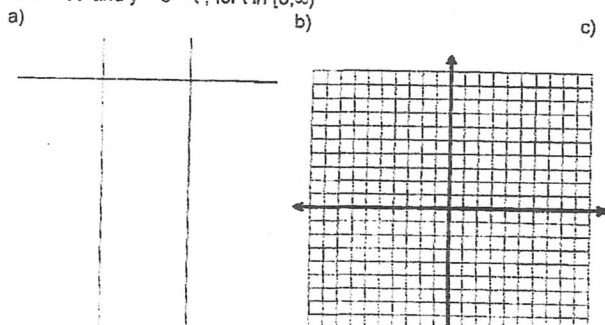
3.  $x = t$  and  $y = |t - 2|$ , for  $t$  in  $[-4, 4]$



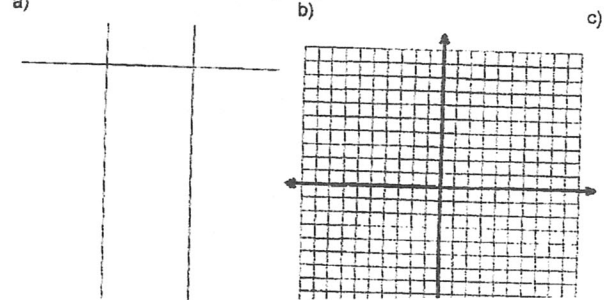
4.  $x = 5 - t^2$  and  $y = t - 3$ , for  $t$  in  $[-2, 3]$



5.  $x = \sqrt{t}$  and  $y = 5 - t$ , for  $t$  in  $[0, \infty)$



6.  $x = t^2 - 4$  and  $y = 2t^2$ , for  $t$  in  $(-\infty, \infty)$

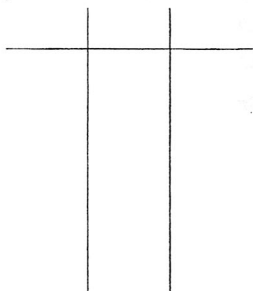


For the following parametric equation:

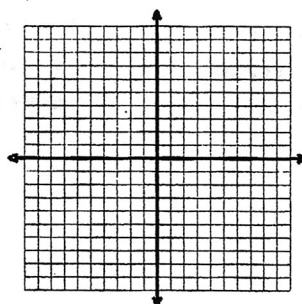
- Make a table of values
- Draw the graph and show the orientation.
- Write the rectangular equation.

$x = t^2 - 9$  and  $y = t^2$ , for  $t$  in  $(-\infty, \infty)$

a)



b)

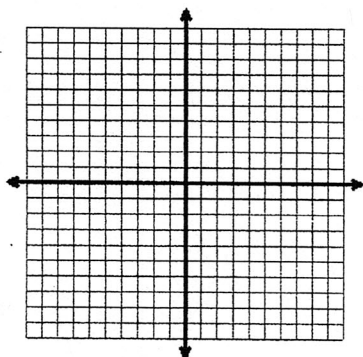


c)

1. Complete the table and graph the following parametric equations for  $t$  in  $[0, 2\pi]$ .

$x = 5\cos t$   
 $y = 5\sin t$

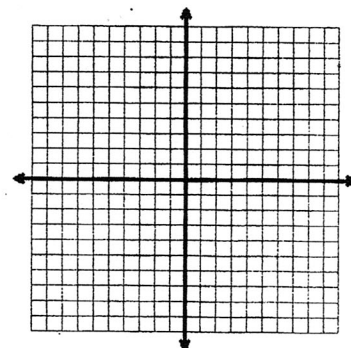
t	x	y
0		
$\frac{\pi}{2}$		
$\pi$		
$\frac{3\pi}{2}$		
$2\pi$		



2. Complete the table and graph the following parametric equations for  $t$  in  $[0, 2\pi]$ .

$x = 2 + 3\cos t$   
 $y = 2 + 6\sin t$

t	x	y
0		
$\frac{\pi}{2}$		
$\pi$		
$\frac{3\pi}{2}$		
$2\pi$		



Type of Curve: \_\_\_\_\_

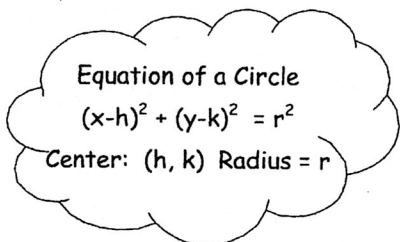
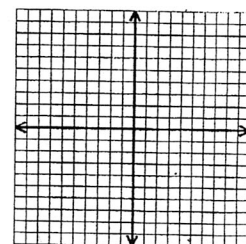
Find the rectangular equation.

Type of Curve: \_\_\_\_\_

Find the rectangular equation.

For the following parametric equations, sketch a complete graph showing your scale on the axes.

$x = 4t - 2$   $y = 8t^2$  for  $t$  in  $[-1, 2]$



Graph the parametric equation, showing orientation. Then eliminate the parameter. State the restrictions on  $x$  and  $y$ .

1.  $x = t - 3$

$y = 2t + 4$

$0 \leq t \leq 2$

2.  $x = t + 2$

$y = \sqrt{t}$

$t \geq 0$

3.  $x = \sqrt{t} + 4$

$y = \sqrt{t} - 4$

$0 \leq t \leq 9$

4.  $x = 2t - 4$

$y = 4t^2$

$-4 \leq t \leq 4$

5.  $x = \cos^2\theta$

$y = \sin^2\theta$

$0 \leq \theta \leq 2\pi$

6.  $x = 2\cos\theta$

$y = 6\sin\theta$

$0 \leq \theta \leq 2\pi$

7.  $x = 3 + 2\cos\theta$

$y = -1 + 2\sin\theta$

$0 \leq \theta \leq 2\pi$

8.  $x = 3\sec\theta$

$y = 4\tan\theta$

$0 \leq \theta \leq 2\pi$